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(54) **Interpolation filter**

(57)   An interpolator for a discrete time signal is provided, which performs the steps of identifying a plurality of interpolation points which interpolation points are symmetrically temporarily displaced about a central reference point of a plurality of discrete time samples of a signal for which interpolated signal samples are to be calculated at the interpolation points, determining a plurality of impulse response coefficients appertaining to the interpolating function at each corresponding sampling time of the said plurality of signal samples, and contemporaneously calculating a plurality of interpolated discrete time signal samples corresponding to the interpolation points in combination with a pre-addition of at least one pair of discrete time signal samples.
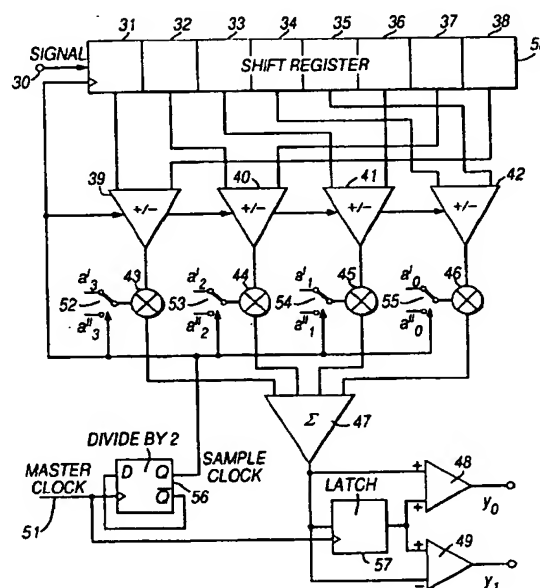

Fig. 4

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Description

The present invention relates to digital signal processors which operate to interpolate discrete time signals.

As is well known to those skilled in the art, interpolation is a process wherein a value of a function or signal is estimated at a particular point from values of the function or signal already known. In the context of digital signal processing, interpolation is used to increase a sample rate of a signal represented by discrete time signal samples. The discrete time signal samples representative of the signal are typically displaced in time at regular intervals in accordance with a sampling interval of the signal. The sampling interval, and therefore the sampling rate, are determined by the bandwidth of frequencies from which the signal is comprised as determined by Nyquist Sampling Theory well known to those skilled in the art. Therefore, to increase the sampling rate of the discrete time signal, the signal must be interpolated to provide signal samples at intermediate time displacements between the known signal samples.

Known interpolators of discrete time signals embody Finite Impulse Response Filters, wherein known signal samples are fed to a shift register embodied therein and, in accordance with the values of the signal samples present in the shift register, an interpolation process is provided to generate interpolated signal samples at intermediate time displacements. Interpolated signal samples are calculated by convolving the discrete time signal samples held in the shift register with an impulse response representative of the interpolating function.

A known technique for providing a reduction in complexity of such interpolating filters or indeed of any Finite Impulse Response (FIR) filter wherein the impulse response of the filter has a linear phase is known as 'pre-addition'. A characteristic of a linear phase impulse response is that coefficients of the impulse response have a symmetrical distribution about a centre or reference point. A pre-addition structure utilises the symmetry of the linear phase impulse response which is characterised in that each impulse response coefficient a_i is equal to an impulse response coefficient in an equivalent temporal position about a centre or reference coefficient point. Thus for a linear phase impulse response $a_i = a_{-i}$. The pre-addition structure operates to provide a reduced complexity interpolator by summing pairs of discrete time signal samples held within the shift register of the FIR interpolator, before multiplication by impulse response coefficients wherein the pairs of signal samples correspond to symmetrical pairs of impulse response coefficients of the interpolator, or, in other words, where a_i is equal to a_{-i}.

FIGURE 1 presents an illustrative example of a Finite Impulse Response interpolating filter, with a pre-addition structure. In Figure 1, a shift register 1' is shown to be comprised of eight stages 1, 2, 3, 4, 5, 6, 7, 8, fed with discrete time signal samples via a conductor 9. Also shown in Figure 1 is a set of lines 10, 11, 12, 13, 14, 15, 16, 17, each of which is representative of a discrete reference corresponding in time to the relative position of each of the stages 1, 2, 3, 4, 5, 6, 7, 8, of the shift register 1'. Each of the lines, 10, 11, 12, 13, 14, 15, 16, 17, is shown with a signal sample represented as x_i where i represents the relative position in time of the impulse response of the filter with respect to a central or reference point x_0. Also shown in Figure 1, is a conceptual line 18, representative of an interpolating function. The interpolating function 18, is shown to be symmetrical about the central or reference position y_0, which corresponds to a temporal position of a interpolated signal sample to be calculated. As a result of the symmetry of the interpolating function 18, each of the coefficients representative of the impulse response of the interpolating function is symmetrical as aforementioned in that $a_1 = a_{-1}$, $a_2 = a_{-2}$ and so forth such that $a_i = a_{-i}$. Connected to the shift register 1' is a plurality of summing amplifiers 19, 20, 21, 22 which operate to form a pre-addition of discrete time signal samples for corresponding coefficients of the impulse response of the interpolation function 18 which are symmetrical in that $a_i = a_{-i}$. At an output of each of the summing amplifiers 19, 20, 21, 22 the pre-addition sum is subsequently scaled by corresponding composite coefficient a_i of the interpolating function by a corresponding plurality of multipliers 23, 24, 25, 26. Each output from the multipliers 23, 24, 25, 26, is fed to a final summer 27, which operates to sum the respective outputs from the multipliers to produce the interpolated signal sample y_0 at a conductor 28.

The pre-addition structure, as illustrated by the Finite Impulse Response interpolating filter shown in Figure 1, therefore provides a saving in the order of a factor of two in complexity, cost and time for interpolation. This saving is achieved by pre-adding each pair of discrete time signal samples X_i associated with a corresponding pair of symmetrical impulse response coefficients a_i and multiplying these pre-added samples by one of the pair of corresponding impulse response coefficients. The result of such multiplications for each pair of discrete time signal samples are thereafter summed to form an interpolated signal sample.

An interpolating Finite Impulse Response filter, which is provided with a pre-addition structure as hereinbefore described, would have an advantage of a reduced complexity implementation. However, provision of a pre-addition structure is conditioned on the impulse response of the interpolating filter being a linear phase response and thereby possessing a necessary symmetry. Although an interpolation function of an interpolating filter may exhibit a linear phase, an impulse response representative of a sampled version of the interpolation function may not be symmetrical about an interpolation point of the function, corresponding to a temporal position of a interpolated sample, preventing the use of a pre-addition structure.

It is an object of the present invention to provide a reduced complexity interpolator.

According to the present invention, there is provided a method of interpolating a discrete time signal comprising

steps of;

(i) identifying a plurality of interpolation points which interpolation points are substantially equally temporarily displaced with respect to at least one point of a plurality of discrete time samples of a signal for which interpolated signal samples are to be calculated at the said interpolation points;

(ii) for each of the interpolation points, determining a plurality of impulse response coefficients appertaining to the interpolating function at each corresponding sampling time of the said plurality of signal samples, and

(iii) contemporaneously calculating the interpolated signal samples corresponding to the interpolation points in combination with sum and difference operations of at least one pair of impulse response coefficients, and at least one pair of discrete time signal samples.

By selecting a plurality of interpolating points, each of which is displaced in time by an equal amount from a central reference point, the calculation of interpolated signal samples corresponding to the interpolation points may be substantially reduced by utilising a relative symmetry of the impulse response coefficients associated with respective interpolation points.

One embodiment of the present invention will now be described by way of example only, with reference to the accompanying drawings wherein;

FIGURE 2 is a representation of two versions of an interpolation function with associated coefficients corresponding to two interpolating points for an odd order interpolator.

FIGURE 3 is a representation of two versions of an interpolation function with associated coefficients corresponding to two interpolating points, for an even order interpolator, and

FIGURE 4 is a schematic block diagram of an even order interpolating filter.

A Finite Impulse Response Filter (FIR) which operates to convolve an impulse response of the filter with a discrete time signal may be an even order filter or an odd order filter. An even order filter comprises an impulse response with an even number of impulse response coefficients whereas an odd order filter embodies an impulse response with an odd number of impulse coefficients. An odd order impulse response is characterised by a fact that a discrete time sample will lie at a central point of the impulse response of the filter. An even order impulse response is characterised in that a sample does not lie at a central point of the impulse response of the filter.

An illustrative representation of an interpolation function in continuous time form is shown in Figure 2. In Figure 2 two versions of the interpolating function are shown 29a, 29b, which correspond to a calculation of two interpolated signal samples y_0 y_1. Also shown in Figure 2, are a plurality of discrete time sampling points represented by lines which are labelled with x_i where index i is in the range -3 to 3. In this case an interpolator for interpolating the signal sample is an odd order interpolator. At each of the discrete time sampling points x_{-3}, x_{-2}, x_{-1}, x_0, x_1, x_2, x_3, a corresponding sample of the interpolating functions 29a, 29b is shown by impulse response coefficient a_i where i is also in the range -3 to 3. The impulse response coefficients, a_i form an impulse response corresponding to a discrete time representation of the interpolating function 29a, 29b which serve to form interpolated samples y_0, y_1. As a result of the fact that the interpolated sampling points y_i y_1 are temporarily displaced with respect to the discrete time sample instant xo by an equal amount, the impulse response coefficient for each of the interpolating points y_1, y_0 are substantially the same, in that the coefficient a_i where i is -3 to 3 are the same for a calculation of an interpolating sample at the interpolation point y_0 as they are for the interpolating point Y_1. The coefficients for calculation of respective interpolated samples y_0 y_1 are therefore symmetrical. It is this symmetry between calculation of interpolated samples that provides a saving in computation of the interpolated samples at points y_0 y_1, in accordance with the following method.

In the example shown in Figure 2, the interpolation point y_0 corresponds to a temporal position, which is a quarter of the sampling interval earlier than a current input signal sample position corresponding to x_0.

The interpolator point y_1 corresponds to a temporal position, which is a quarter of the sampling interval later than the current input signal sample corresponding to x_0. The coefficients for the interpolation point are shown on the diagram as a_i with $i \in \{-3...3\}$. Thus

$$y_0 = \sum_{i=-3}^{3} a_i x_i$$

and, more generally for an odd order interpolator:-

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$$y_0 = \sum_{i=-k}^{k} a_i x_i$$

where the order of the filter is $2k+1$. Inspection of Figure 2 shows a form of symmetry between the coefficients of the interpolation point for early samples y_0 and that for the interpolation point for late samples Y_1. Thus we can see that

$$y_1 = \sum_{i=-3}^{3} a_{-i} x_i$$

and, more generally:-

$$y_1 = \sum_{i=-k}^{k} a_{-i} x_i.$$

We may express the above as follows:-

$$y_0 = a_0 x_0 + \sum_{i=-k}^{1} a_i x_i + \sum_{i=1}^{k} a_i x_i = a_0 x_0 + \sum_{i=1}^{k} \left(a_i x_i + a_{-i} x_{-i} \right)$$

and

$$y_1 = a_0 x_0 + \sum_{i=-k}^{1} a_{-i} x_i + \sum_{i=1}^{k} a_{-i} x_i = a_0 x_0 + \sum_{i=1}^{k} \left(a_{-i} x_i + a_i x_{-i} \right)$$

Now form

$$\alpha = \frac{y_0 + y_1}{2} = a_0 x_0 + \frac{1}{2} \sum_{i=1}^{k} \left(a_i x_i + a_{-i} x_{-i} + a_{-i} x_i + a_i x_{-i} \right)$$

(1)

$$\alpha = a_0 x_0 + \frac{1}{2} \sum_{i=1}^{k} \left(a_i + a_{-i} \right)\left(x_i + x_{-i} \right) = \sum_{i=0}^{k} a_i'\left(x_i + x_{-i} \right) \qquad (1)$$

where $a_i' = \begin{cases} i = 0 & \dfrac{a_0}{2} \\[2mm] i > 0 & \dfrac{a_i + a_{-i}}{2} \end{cases}$

and

(2)

$$\beta = \frac{y_0 - y_1}{2} = \frac{1}{2} \sum_{i=1}^{k} \left(a_i x_i + a_{-i} x_{-i} - a_{-i} x_i - a_i x_{-i} \right)$$

$$\beta = \frac{1}{2} \sum_{i=1}^{k} \left(a_i - a_{-i} \right)\left(x_i - x_{-i} \right) = \sum_{i=1}^{k} a_0''\left(x_i - x_{-i} \right) \qquad (2)$$

where $a_i'' = \dfrac{a_i - a_{-i}}{2}$

The interpolated signal samples corresponding to the sampling points y_0, y_1 are thereafter calculated in accord-

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ance with the sum of α and β for y_0, and a difference between α and β for y_1 as represented by equations (5) and (6).

$$y_0 = \alpha + \beta \tag{5}$$

$$y_1 = \alpha - \beta \tag{6}$$

A representation of two versions of the interpolation function for calculation of two interpolated samples at positions y_0 y_1 for an even order impulse response is shown in Figure 3 where parts or elements also represented in Figure 2 bear identical numerical or alpha numeric designations. In Figure 3, the interpolating function 29a, 29b, are shown to be sampled in accordance with a plurality of discrete time sampling positions xi as for Figure 2, but unlike Figure 2, there are now eight sampling positions and correspondingly the impulse coefficients a_i for each of these even number of sampling positions are adjusted in accordance with the temporal position of each of the discrete time sampling points xi where now i is in the range -4 to +3. In this case we have

$$y_0 = \sum_{i=-k}^{k-1} a_i x_i$$

for a filter of order $2k(k = 3$ in Figure 3). With a symmetry similar to that described earlier, we have

$$y_1 = \sum_{i=-k}^{k-1} a_{-(i+1)} x_i.$$

We can express the above as:-

$$y_0 = \sum_{i=-k}^{-1} a_i x_i + \sum_{i=0}^{k-1} a_i x_i = \sum_{i=0}^{k-1} (a_i x_i + a_{-(i+1)} x_{-(i+1)})$$

and

$$y_1 = \sum_{i=-k}^{-1} a_{-(i+1)} x_i + \sum_{i=0}^{k-1} a_{-(i+1)} x_i = \sum_{i=0}^{k-1} (a_{-(i+1)} x_i + a_i x_{-(i+1)})$$

Now form

$$\alpha = \frac{y_0 + y_1}{2} = \frac{1}{2} \sum_{i=1}^{k} (a_i x_i + a_{-(i+1)} x_{-(i+1)} + a_{-(i+1)} x_i + a_i x_{-(i+1)}) \tag{3}$$

$$\alpha = \frac{1}{2} \sum_{i=1}^{k} (a_i + a_{-(i+1)})(x_i + x_{-(i+1)}) = \sum_{i=1}^{k} a_i^{'}(x_i + x_{-(i+1)})$$

where

$$a_i^{'} = \frac{a_i + a_{-(i+1)}}{2}$$

and

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$$\beta = \frac{y_0 - y_1}{2} = \frac{1}{2} \sum_{i=1}^{k} (a_i x_i + a_{-(i+1)} x_{-(i+1)} - a_{-(i+1)} x_i - a_i x_{-(i+1)}) \tag{4}$$

$$\beta = \frac{1}{2} \sum_{i=1}^{k} (a_i - a_{-(i+1)})(x_i - x_{-(i+1)}) = \sum_{i=1}^{k} \bar{a}_0 (x_i - x_{-(i+1)})$$

where

$$\bar{a}_i = \frac{a_i - a_{-(i+1)}}{2}$$

Thus, as for the odd interpolator case, we have $y_0 = \alpha + \beta$ and $y_1 = \alpha - \beta$ where α and β are generated with only k multiplications each.

Consider now a hardware (or software) implementation which uses the final expressions to generate α and β. For α we perform a pre-addition over the samples (for convenience, pre-adding the centre sample to itself) and multiply by the taps a'_i, $i \in \{0...k\}$. Thus we perform $k + 1$ multiplications to form α. For β we perform a pre-subtraction over the samples (in this case the subtraction of the centre sample from itself gives zero so this term is ignored) and multiply the taps by $\bar{a}_i$, $i \in \{1...k\}$. Thus we perform k multiplications to form β. In total we perform $2k+1$ multiplications to generate α and β. Given α and β we can easily derive y_0 and y_1 since $y_0 = \alpha + \beta$ and $y_1 = \alpha - \beta$. as hereinbefore explained. Thus we have generated both y_0 and y_1 by performing only $2k + 1$ multiplications whereas in the ordinary implementation, they would require $2k + 1$ multiplications each. Thus this approach leads to a halving of the number of multiplications.

A corresponding implementation of a FIR interpolating filter for performing the interpolation of an even order impulse response is shown in Figure 4. The above approach for generating interpolated samples may be implemented in a variety of efficient hardware or software embodiments. Figure 4, shows the architecture for an 8th order ($k = 4$) interpolator hardware embodiment of an interpolater which operates in accordance with the interpolation method hereinbefore described for an even order interpolator.

In Figure 4 a discrete time signal is fed from a conductor 30, to a shift register 58, comprised of eight stages 31, 32, 33, 34, 35, 36, 37, 38. Also shown in Figure 4, are four controllable adder subtractors 39, 40, 41, 42. A first adder/subtractor 39, is connected to a first stage 31, and to the eighth stage 38, of the shift register 58. Adder subtractor 40, is connected to a second stage 32, and to the seventh stage 37, of the shift register 58, whereas the third adder subtractor 41, is connected to the third stage 33, and to the sixth stage 36, of the shift register 58. Finally, the fourth adder subtractor 42, is connected to the fourth stage 34, and to the fifth stage 35, of the shift register 58. An output of each of the four adder subtractors 39, 40, 41, 42, is respectively connected to an input of one of four multipliers 43, 44, 45 and 46. An output of each of the four multipliers 43, 44, 45, 46, is connected to an input of an adder 47. An output of the adder 47, is connected to one positive input of a first arithmetic unit 48. The output form the adder 47, is also connected to a negative input of a second arithmetic unit 49. The output from the adder 47 is also connected to an input of a latch 50. An output of the latch 50, is connected to a second positive input of the first arithmetic unit 48 and a second positive input of the second arithmetic unit 49. A clock input of the latch is connected to a master clock via a conductor 51, which is also connected to a clock input of a divide by two D-type flip-flop 56. A Q output of the D-type flip-flop is connected to a clock input of the shift register 58, and respectively to a control input of each of the adder subtractors 39, 40, 41, 42. The Q output from the D-type flip-flop is also connected to a control input of each of four switches 52, 53, 54, 55. An output from each of the switches 52, 53, 54, 55, is respectively connected to a second input of each of the multipliers 43, 44, 45, 46.

In Figure 4, the shift register 58. clock in signals samples, x_i, where $i \in \{-4...3\}$. The master clock runs at twice the sample rate. The sample clock which drives the shift register 58, is derived from a version of the master clock, which is divided by two by the divide by 2 flip-flop 56, to produce a sample clock representative of the master clock, but with half the frequency. On the rising edge of the sample clock a new signal sample is shifted into the shift register.

The four adder/subtractors 39,40,41,42, are arranged to out put $x_i \pm x_{-(i+1)}$ depending on the state of the sample clock. An adder/subtractor adds its inputs together if configured to add (sample clock high) and subtracts the right hand input from the left hand input if configured to subtract (sample clock low). The rightmost adder/subtractor provides the output for $i = 0$ and the index increases from right to left. For each position, i, the output is multiplied by either a'_i (if the adder/subtractor is adding) or $\bar{a}_i$ (if the adder/subtractor is subtracting). The choice of a'_i or $\bar{a}_i$ is according to the state of the switches.

During the interval between successive cycles of the sample clock, there are two phases of the operation of the master clock. During the first phase the sample clock is high and the circuit is arranged to compute the current value of

α. The second phase begins with the next rising edge of the master clock. This clocks the value of α into the latch 57, and sets the sample clock low. In this state the circuit is arranged to compute the value of β. Once the computed value of β becomes valid (towards the end of the master clock cycle), the outputs of the adder and subtractor provide $y_0 = \alpha + \beta$ and $y_1 = \alpha - \beta$ respectively. Thus, in this implementation, the two outputs are generated contemporaneously.

As will be appreciated by those skilled in the art various modifications may be made to the arrangement of the interpolating filter without departing from the scope of the invention, in particular it may be appropriate to interpolate several values between the regular input samples. There may, nevertheless, remain a symmetry between the coefficients needed to generate half of a subset of these samples and the other half of that subset. The principles hereinbefore described may therefore be extended to this case by generating the same filtering operations to generate each suitable pair of interpolated values. Note, however, that a further saving is possible in this case because the required pre-addition and pre-subtraction operations are common to all required interpolated values. Thus these operations need only be performed once.

Claims

1. A method of interpolating a discrete time signal comprising steps of;

   (i) identifying a plurality of interpolation points which interpolation points are substantially equally temporarily displaced with respect to at least one of a plurality of discrete time samples of a signal for which interpolated signal samples are to be calculated at the said interpolation points;
   (ii) for each of the interpolation points, determining a plurality of impulse response coefficients appertaining to the interpolating function at each corresponding sampling time of the said plurality of signal samples, and
   (iii) contemporaneously calculating the interpolated signal samples corresponding to the interpolation points in combination with sum and difference operations of at least one pair of impulse response coefficients, and at least one pair of discrete time signal samples.

2. A method of interpolating a discrete time signal as claimed in Claim 1, wherein the said plurality of impulse response coefficients of the interpolation function determined in step (ii) of the said method are substantially the same for each of the said interpolation points.

3. A method of interpolating a discrete time signal as claimed in Claim 1 and 2, wherein step (iii) of the said method comprises the steps of;

   (iv) forming first intermediate data α, in accordance with at least one summed pair of discrete time signal samples, in combination with at least one summed pair of impulse response coefficients;
   (v) forming second intermediate data β, in accordance with a difference between at least one pair of discrete time signal samples, in combination with a difference between at least one pair of impulse response coefficients, and
   (vi) calculating the interpolated signal samples in accordance with a difference between the first intermediate data α, and the second intermediate data β, and a sum of the first intermediate data α, and the second intermediate data β.

4. A method of interpolating a discrete time signal as claimed in Claim 3, wherein each signal sample of the said pair of at least one pair of signal samples in step (iv) of the said method for forming the first intermediate sum α, is substantially equally temporally displaced with respect to the central reference point, and the said at least one pair of impulse response coefficients are representative of samples of the interpolation function at temporal positions appertaining the temporal displacement of the said at least one pair of signal samples with respect to the central reference point.

5. A method of interpolating a discrete time signal as claimed in Claim 3 or 4, wherein each signal sample of the said pair of at least one pair of signal samples in step (v) of the said method for forming the second intermediate sum β, is substantially equally temporally displaced with respect to the central reference point, and the said at least one pair of impulse response coefficients are representative of samples of the interpolation function at temporal positions appertaining the temporal displacement of the said at least one pair of signal samples with respect to the central reference point.

6. A method of interpolating a discrete time signal as claimed in Claim 3 to 5, wherein each of the said pairs of signal

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samples in step (iv) of the said method for forming the first intermediate sum α, are substantially equal to the said pairs of signal samples in step (v) of the said method for forming the second intermediate sum β.

7. A method of interpolating a discrete time signal as claimed in Claim 3 to 6, wherein each the said pairs of impulse response coefficients in step (iv) of the said method for forming the first intermediate sum α, are substantially the same as the said pairs of impulse response coefficients in step (v) of the said method for forming the second intermediate sum β.

8. A method of interpolating a discrete time signal as claimed in Claim 7, wherein the plurality of discrete time signal samples is an odd number of signal samples represented as $2k + 1$, where k is an integer, and step (iv) of the said method for forming the first intermediate sum α, is calculated in accordance with equation (1) as follows;

$$\alpha = a_0 x_0 + \frac{1}{2} \sum_{i=1}^{k} (a_i + a_{-i})(x_i + x_{-i}) = \sum_{i=0}^{k} a'_i (x_i + x_{-i}) \tag{1}$$

wherein

$$a'_i = \begin{cases} i = 0 & \dfrac{a_0}{2} \\ i > 0 & \dfrac{a_i + a_{-i}}{2} \end{cases}$$

and where a_i represents the i-th impulse response coefficient of the interpolation function, and x_i represents the i-th signal sample, and step (v) of the said method for forming the second intermediate sum β, is calculated in accordance with equation (2) as follows;

$$\beta = \frac{1}{2} \sum_{i=1}^{k} (a_i - a_{-i})(x_i - x_{-i}) = \sum_{i=1}^{k} a''_0 (x_i - x_{-i}) \tag{2}$$

where

$$a''_i = \frac{a_i - a_{-i}}{2} .$$

9. A method of interpolating a discrete time signal as claimed in Claim 7, wherein the plurality of discrete time signal samples is an even number of signal samples represented as $2k$, where k is an integer, and step (iv) of the said method for forming the first intermediate sum α, is calculated in accordance with equation (3) as follows; for

$$\alpha = \frac{1}{2} \sum_{i=1}^{k} (a_i + a_{-(i+1)})(x_i + x_{-(i+1)}) = \sum_{i=1}^{k} a'_i (x_i + x_{-(i+1)}) \tag{3}$$

where

$$a'_i = \frac{a_i + a_{-(i+1)}}{2}$$

and where a_i represents the i-th impulse response coefficient of the interpolation function, and x_i represents the i-th signal sample, and step (v) of the said method for forming the second intermediate sum β, is calculated in accordance with equation (4) as follows;

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$$\beta = \frac{1}{2} \sum_{i=1}^{k} (a_i - a_{-(i+1)})(x_i - x_{-(i+1)}) = \sum_{i=1}^{k} a_0^{''}(x_i - x_{-(i+1)}) \tag{4}$$

where

$$a_i^{''} = \frac{a_i - a_{-(i+1)}}{2}.$$

10. A method of interpolating a discrete time signal as claimed in Claims 8 or 9, wherein the plurality of interpolation points are two interpolation points, designated y_0 and y_1, and wherein $y_0 = \alpha + \beta$ and $y_1 = \alpha - \beta$.

11. An interpolating filter for generating a plurality of interpolated signal samples of a discrete time signal, comprises a shift register, which shift register serves to store samples of the discrete time signal fed thereto, a plurality of switchable adder/subtractors connected to the said shift register, each of which said adder/subtractors operates in a first mode to add a pair of signal samples communicated from the shift register, and in a second mode to subtract the pair of signal samples from each other, and which adder/subtractors are switched between first and second modes in accordance with a control signal, a plurality of scaling means, each of which scaling means operates to scale signals communicated from an output of the said adder/subtractors with data communicated from one of a plurality of controllable switches, the data fed from each controllable switch being either a sum of two interpolating coefficients in a first position of the said switch, or a difference between the interpolating coefficients in a second position of the said switch, which position of the switch is controlled in accordance with the said control signal, a summer connected to an output of each of the plurality of scaling means, which summer operates to sum scaled outputs from the said adder/subtractors, and an interpolating controller which operates to generate the control signal and the interpolated signal samples in accordance with sum and difference operations on signals provided at an output of the summer.

12. An interpolating filter as hereinbefore described with reference to Figures 2, 3, and 4.
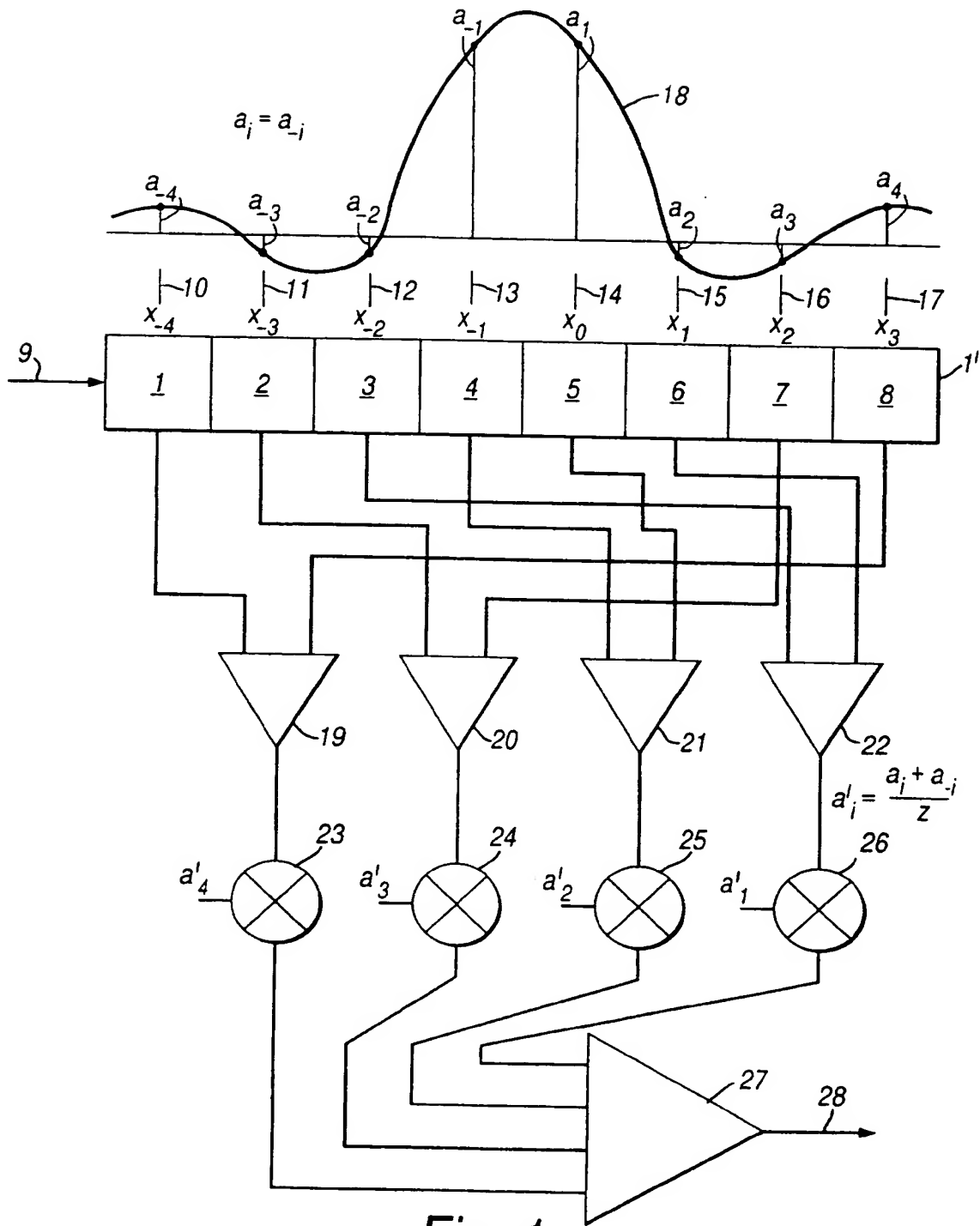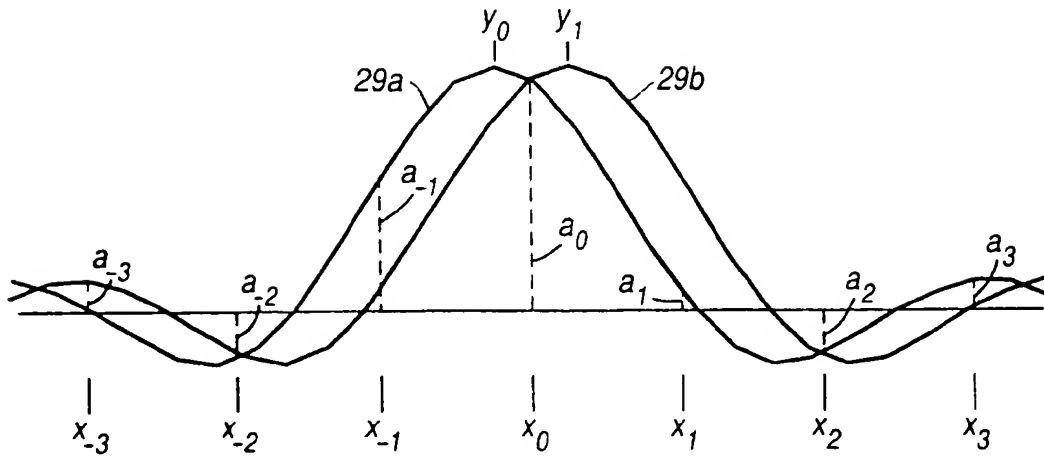
$a_i = a_{-i}$
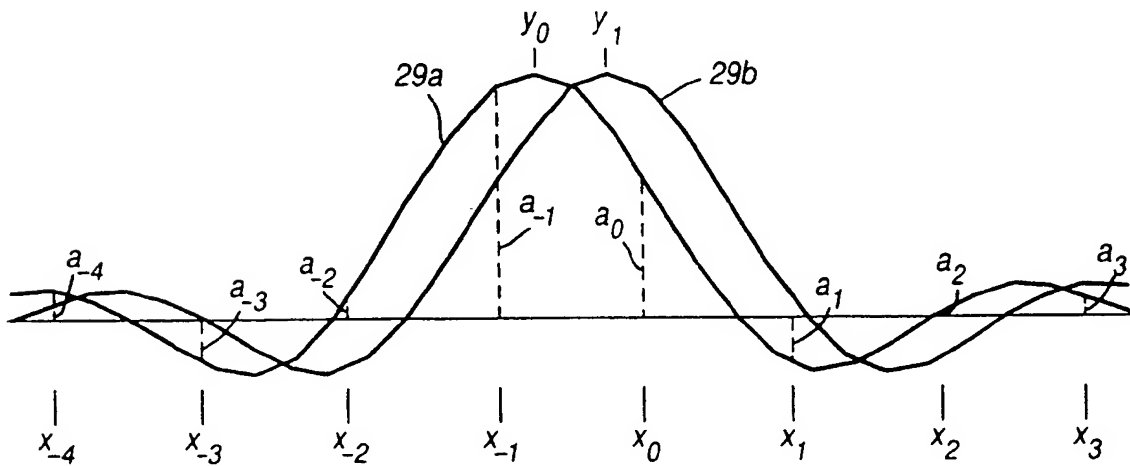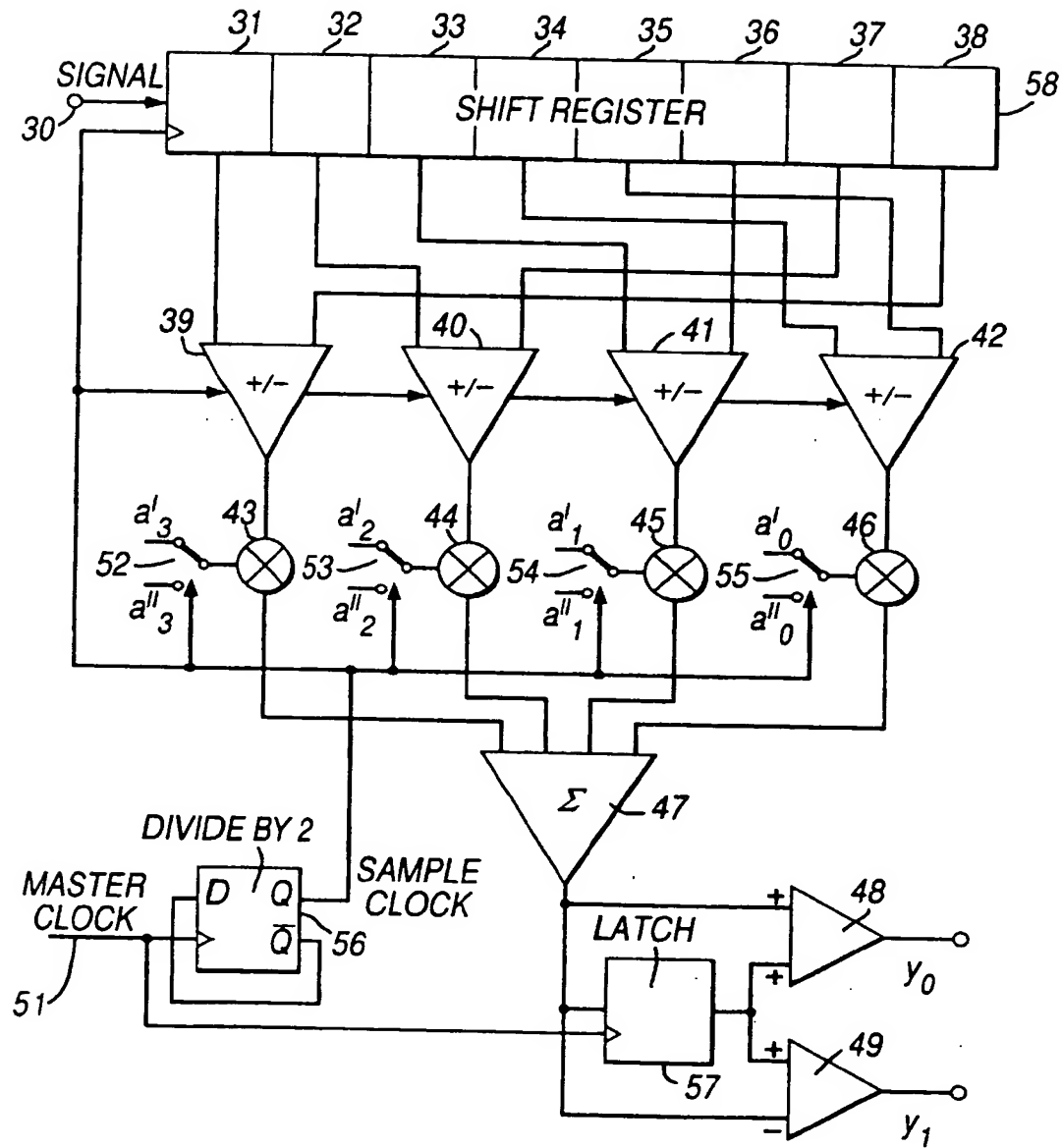
$$a'_i = \frac{a_i + a_{-i}}{z}$$

Fig. 1

Fig. 2

Fig. 3

Fig. 4

PATENT COOPERATION TREATY

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From the INTERNATIONAL SEARCHING AUTHORITY

NOTIFICATION OF TRANSMITTAL OF
THE INTERNATIONAL SEARCH REPORT
OR THE DECLARATION

(PCT Rule 44.1)

To:
STERNE, KESSLER, GOLDSTEIN & FOX P.L.L.C. Attn. Featherstone, Donald J. 1100 New York Avenue, N.W. Suite 600 Washington, D.C. 20005-3934 UNITED STATES OF AMERICA

Date of mailing *(day/month/year)*	18/09/2002

Applicant's or agent's file reference	FOR FURTHER ACTION	See paragraphs 1 and 4 below
1857.033PC00		

International application No.	International filing date *(day/month/year)*	11/01/2002
PCT/US 02/ 00556		

Applicant
SILICON VALLEY GROUP, INC.

1. [X] The applicant is hereby notified that the International Search Report has been established and is transmitted herewith.

   Filing of amendments and statement under Article 19:
   The applicant is entitled, if he so wishes, to amend the claims of the International Application (see Rule 46):

   When? The time limit for filing such amendments is normally 2 months from the date of transmittal of the International Search Report; however, for more details, see the notes on the accompanying sheet.

   Where? Directly to the   International Bureau of WIPO
   34, chemin des Colombettes
   1211 Geneva 20, Switzerland
   Fascimile No.: (41-22) 740.14.35

   DOCKETED

   Response to Search (11/18/02) 10-18-02

   For more detailed instructions, see the notes on the accompanying sheet.

   Rule 156 I 05 (12/18/02) 11-18-02

2. [ ] The applicant is hereby notified that no International Search Report will be established and that the declaration under Article 17(2)(a) to that effect is transmitted herewith.

   Australia DDS (3/8/03) 2-18-03

3. [ ] **With regard to the protest** against payment of (an) additional fee(s) under Rule 40.2, the applicant is notified that:

   [ ] the protest together with the decision thereon has been transmitted to the International Bureau together with the applicant's request to forward the texts of both the protest and the decision thereon to the designated Offices.

   [ ] no decision has been made yet on the protest; the applicant will be notified as soon as a decision is made.

4. **Further action(s):**   The applicant is reminded of the following:

   Shortly after **18 months** from the priority date, the international application will be published by the International Bureau. If the applicant wishes to avoid or postpone publication, a notice of withdrawal of the international application, or of the priority claim, must reach the International Bureau as provided in Rules 90*bis*.1 and 90*bis*.3, respectively, before the completion of the technical preparations for international publication.

   Within **19 months** from the priority date, a demand for international preliminary examination must be filed if the applicant wishes to postpone the entry into the national phase until 30 months from the priority date (in some Offices even later).

   Within **20 months** from the priority date, the applicant must perform the prescribed acts for entry into the national phase before all designated Offices which have not been elected in the demand or in a later election within 19 months from the priority date or could not be elected because they are not bound by Chapter II.

Name and mailing address of the International Searching Authority	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL-2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Shantisaroop Pherai

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NOTES TO FORM PCT/ISA/220

These Notes are intended to give the basic instructions concerning the filing of amendments under article 19. The Notes are based on the requirements of the Patent Cooperation Treaty, the Regulations and the Administrative Instructions under that Treaty. In case of discrepancy between these Notes and those requirements, the latter are applicable. For more detailed information, see also the PCT Applicant's Guide, a publication of WIPO.

In these Notes, "Article", "Rule", and "Section" refer to the provisions of the PCT, the PCT Regulations and the PCT Administrative Instructions respectively.

INSTRUCTIONS CONCERNING AMENDMENTS UNDER ARTICLE 19

The applicant has, after having received the international search report, one opportunity to amend the claims of the international application. It should however be emphasized that, since all parts of the international application (claims, description and drawings) may be amended during the international preliminary examination procedure, there is usually no need to file amendments of the claims under Article 19 except where, e.g. the applicant wants the latter to be published for the purposes of provisional protection or has another reason for amending the claims before international pbulication. Furthermore, it should be emphasized that provisional protection is available in some States only.

What parts of the International application may be amended?

Under Article 19, only the claims may be amended.

During the international phase, the claims may also be amended (or further amended) under Article 34 before the International Preliminary Examining Authority. The description and drawings may only be amended under Article 34 before the International Examining Authority.

Upon entry into the national phase, all parts of the international application may be amended under Article 28 or, where applicable, Article 41.

When? Within 2 months from the date of transmittal of the international search report or 16 months from the priority date, whichever time limit expires later. It should be noted, however, that the amendments will be considered as having been received on time if they are received by the International Bureau after the expiration of the applicable time limit but before the completion of the technical preparations for international publication (Rule 46.1).

Where not to file the amendments?

The amendments may only be filed with the International Bureau and not with the receiving Office or the International Searching Authority (Rule 46.2).

Where a demand for international preliminary examination has been/is filed, see below.

How? Either by cancelling one or more entire claims, by adding one or more new claims or by amending the text of one or more of the claims as filed.

A replacement sheet must be submitted for each sheet of the claims which, on account of an amendment or amendments, differs from the sheet originally filed.

All the claims appearing on a replacement sheet must be numbered in Arabic numerals. Where a claim is cancelled, no renumbering of the other claims is required. In all cases where claims are renumbered, they must be renumbered consecutively (Administrative Instructions, Section 205(b)).

The amendments must be made In the language In which the International application is to be published.

What documents must/may accompany the amendments?

Letter (Section 205(b)):

The amendments must be submitted with a letter.

The letter will not be published with the international application and the amended claims. It should not be confused with the "Statement under Article 19(1)" (see below, under "Statement under Article 19(1)").

The letter must be In English or French, at the choice of the applicant. However, if the language of the International application Is English, the letter must be In English; If the language of the International application Is French, the letter must be In French.

The letter must indicate the differences between the claims as filed and the claims as amended. It must, in particular, indicate, in connection with each claim appearing in the international application (it being understood that identical indications concerning several claims may be grouped),whether

    (i)   the claim is unchanged;

    (ii)   the claim is cancelled;

    (iii)   the claim is new;

    (iv)   the claim replaces one or more claims as filed;

    (v)   the claim is the result of the division of a claim as filed.

The following examples illustrate the manner in which amendments must be explained in the accompanying letter:

1. [Where originally there were 48 claims and after amendment of some claims there are 51]:
"Claims 1 to 29, 31, 32, 34, 35, 37 to 48 replaced by amended claims bearing the same numbers; claims 30, 33 and 36 unchanged; new claims 49 to 51 added."

2. [Where originally there were 15 claims and after amendment of all claims there are 11]:
"Claims 1 to 15 replaced by amended claims 1 to 11."

3. [Where originally there were 14 claims and the amendments consist in cancelling some claims and in adding new claims]:
"Claims 1 to 6 and 14 unchanged; claims 7 to 13 cancelled; new claims 15, 16 and 17 added." or
"Claims 7 to 13 cancelled; new claims 15, 16 and 17 added; all other claims unchanged."

4. [Where various kinds of amendments are made]:
"Claims 1-10 unchanged; claims 11 to 13, 18 and 19 cancelled; claims 14, 15 and 16 replaced by amended claim 14; claim 17 subdivided into amended claims 15, 16 and 17; new claims 20 and 21 added."

"Statement under article 19(1)" (Rule 46.4)

The amendments may be accompanied by a statement explaining the amendments and indicating any impact that such amendments might have on the description and the drawings (which cannot be amended under Article 19(1)).

The statement will be published with the international application and the amended claims.

It must be in the language in which the international appplication is to be published.

It must be brief, not exceeding 500 words if in English or if translated into English.

It should not be confused with and does not replace the letter indicating the differences between the claims as filed and as amended. It must be filed on a separate sheet and must be identified as such by a heading, preferably by using the words "Statement under Article 19(1)."

It may not contain any disparaging comments on the international search report or the relevance of citations contained in that report. Reference to citations, relevant to a given claim, contained in the international search report may be made only in connection with an amendment of that claim.

Consequence if a demand for international preliminary examination has already been filed

If, at the time of filing any amendments under Article 19, a demand for international preliminary examination has already been submitted, the applicant must preferably, at the same time of filing the amendments with the International Bureau, also file a copy of such amendments with the International Preliminary Examining Authority (see Rule 62.2(a), first sentence).

Consequence with regard to translation of the international application for entry into the national phase

The applicant's attention is drawn to the fact that, where upon entry into the national phase, a translation of the claims as amended under Article 19 may have to be furnished to the designated/elected Offices, instead of, or in addition to, the translation of the claims as filed.

For further details on the requirements of each designated/elected Office, see Volume II of the PCT Applicant's Guide.

PATENT COOPERATION TREATY

PCT

INTERNATIONAL SEARCH REPORT

(PCT Article 18 and Rules 43 and 44)

Applicant's or agent's file reference	**FOR FURTHER**	see Notification of Transmittal of International Search Report
1857.033PC00	**ACTION**	(Form PCT/ISA/220) as well as, where applicable, Item 5 below.

International application No.	International filing date *(day/month/year)*	(Earliest) Priority Date *(day/month/year)*
PCT/US 02/ 00556	11/01/2002	11/01/2001

Applicant
SILICON VALLEY GROUP, INC.

This International Search Report has been prepared by this International Searching Authority and is transmitted to the applicant according to Article 18. A copy is being transmitted to the International Bureau.

This International Search Report consists of a total of _____3_____ sheets.

[X] It is also accompanied by a copy of each prior art document cited in this report.

1. **Basis of the report**

   a. With regard to the **language**, the international search was carried out on the basis of the international application in the language in which it was filed, unless otherwise indicated under this item.

       [ ] the international search was carried out on the basis of a translation of the international application furnished to this Authority (Rule 23.1(b)).

   b. With regard to any **nucleotide and/or amino acid sequence** disclosed in the international application, the international search was carried out on the basis of the sequence listing :

       [ ] contained in the international application in written form.

       [ ] filed together with the international application in computer readable form.

       [ ] furnished subsequently to this Authority in written form.

       [ ] furnished subsequently to this Authority in computer readble form.

       [ ] the statement that the subsequently furnished written sequence listing does not go beyond the disclosure in the international application as filed has been furnished.

       [ ] the statement that the information recorded in computer readable form is identical to the written sequence listing has been furnished

2. [ ] **Certain claims were found unsearchable** (See Box I).

3. [ ] **Unity of invention is lacking** (see Box II).

4. With regard to the **title,**

       [X] the text is approved as submitted by the applicant.

       [ ] the text has been established by this Authority to read as follows:

5. With regard to the **abstract,**

       [X] the text is approved as submitted by the applicant.

       [ ] the text has been established, according to Rule 38.2(b), by this Authority as it appears in Box III. The applicant may, within one month from the date of mailing of this international search report, submit comments to this Authority.

6. The figure of the **drawings** to be published with the abstract is Figure No.   1

       [X] as suggested by the applicant.   [ ] None of the figures.

       [ ] because the applicant failed to suggest a figure.

       [ ] because this figure better characterizes the invention.

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7   H03H17/06   H03M1/20

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7   H03M   H03H

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	EP 0 820 145 A (ROKE MANOR RESEARCH) 21 January 1998 (1998-01-21) the whole document	1,3,4,8, 10,11

[ ] Further documents are listed in the continuation of box C.   [X] Patent family members are listed in annex.

° Special categories of cited documents :

'A' document defining the general state of the art which is not considered to be of particular relevance

'E' earlier document but published on or after the international filing date

'L' document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

'O' document referring to an oral disclosure, use, exhibition or other means

'P' document published prior to the international filing date but later than the priority date claimed

'T' later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

'X' document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

'Y' document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

'&' document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
9 September 2002	18/09/2002

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040. Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Coppieters, C

Form PCT/ISA/210 (second sheet) (July 1992)

1

Patent document cited in search report		Publication date	Patent family member(s)		Publication date
EP 0820145	A	21-01-1998	GB	2315625 A	04-02-1998
			EP	0820145 A2	21-01-1998